To Max or not to Max: Online Learning for Speeding Up Optimal Planning

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Outline

1. Motivation
2. Theoretical Model
3. From Model to Practice
   - Dealing with Model Assumptions
   - Learning
   - Using the Classifier
4. Experimental Evaluation

Motivation

- We want to do domain independent optimal planning, in a time-bounded setting
- Use A^*

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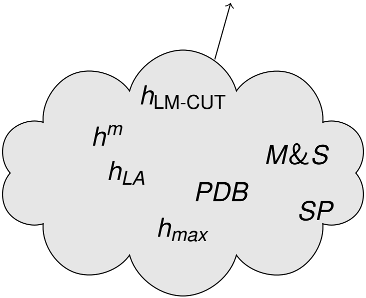

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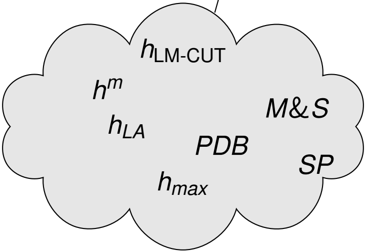

Which heuristic is the best?

Why Settle for One?

- There is no single best heuristic, so why settle only for one?
- We can use the maximum of several heuristics to get a more informative heuristic

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Domain	h_{LA}	$h_{\text{LM-CUT}}$	$\max_h$
airport	25	**38**	36
freecell	**28**	15	22

Number of problems solved in 30 minutes

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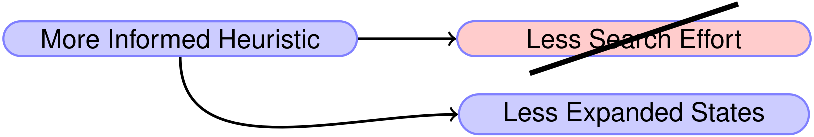
- A more informed heuristic solves less problems — something is rotten in the kingdom of A^*

The Accuracy / Computation Time Tradeoff

More Informed Heuristic → Less Search Effort

The Accuracy / Computation Time Tradeoff



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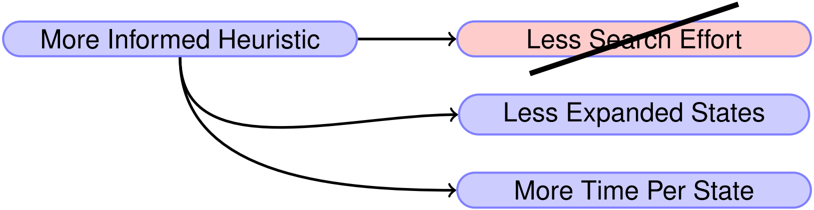

The Accuracy / Computation Time Tradeoff

More Informed Heuristic

~~Less Search Effort~~

Less Expanded States

More Time Per State

$$t_{max_h} = t_{h_{LA}} + t_{h_{\text{LM-CUT}}}$$

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More Informed Heuristic → Less Expanded States

$$t_{max_h} = t_{h_{LA}} + t_{h_{\text{LM-CUT}}}$$ → More Time Per State

Conclusion

A more informed heuristic is not necessarily better

A Simple Observation

- So how can we benefit from multiple heuristics?
- Simple observation: the maximum of several heuristics — is simply the value of one of those heuristics
- This leads to the following idea:
  - Given state s, and heuristics $\{h_1, \ldots, h_n\}$
  - Choose $h_i = \text{ORACLE}(s, \{h_1, \ldots, h_n\})$
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The Oracle

- How do we define ORACLE?
  - Naive answer: use the heuristic which gives the maximum value

$$\text{ORACLE}(s, \{h_1, \ldots, h_n\}) = \operatorname*{argmax}_i h_i(s)$$

  - Why is this naive?
  - Because sometimes the extra time to compute the most informed heuristic is not worth it
  - Example: $h_{\text{LM-CUT}}$ is about 9.4 times slower than h_{LA}

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Our Contributions

- We develop a theoretical model for determining which heuristic is best to compute at each state, in order to minimize search time
- We derive a decision rule from the model, which is used as a target concept for a classifier
- We describe an online learning scheme which uses this classifier during search

Outline

Theoretical Model - Which Heuristic to Compute When?

Assumptions

- State space is a tree
- Single goal state
- Uniform cost actions
- Constant branching factor b
- Perfect knowledge

Two heuristics: h_1 and h_2

- Consistent
- Evaluating h_i takes time t_i

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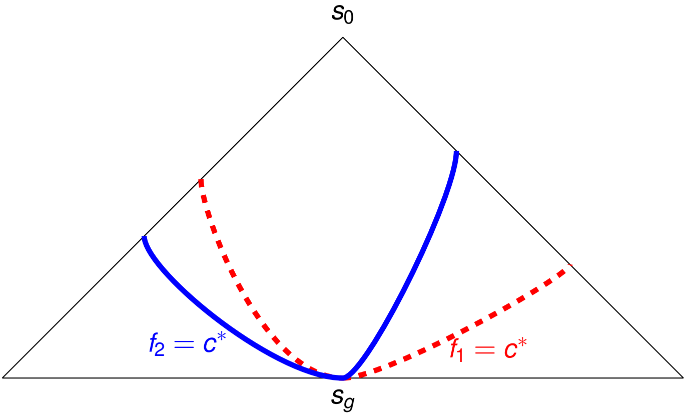

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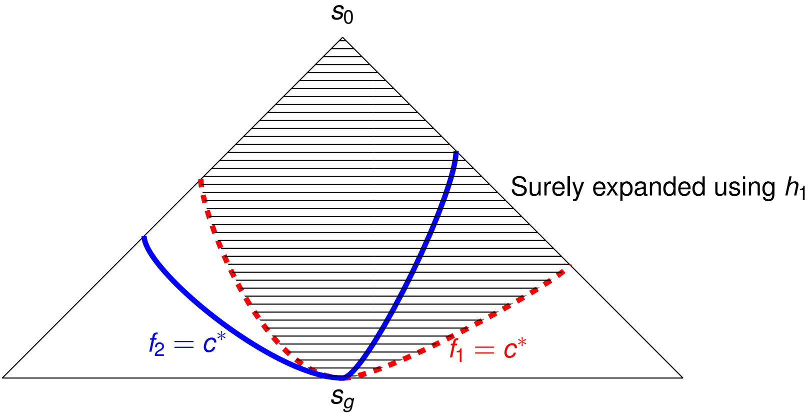

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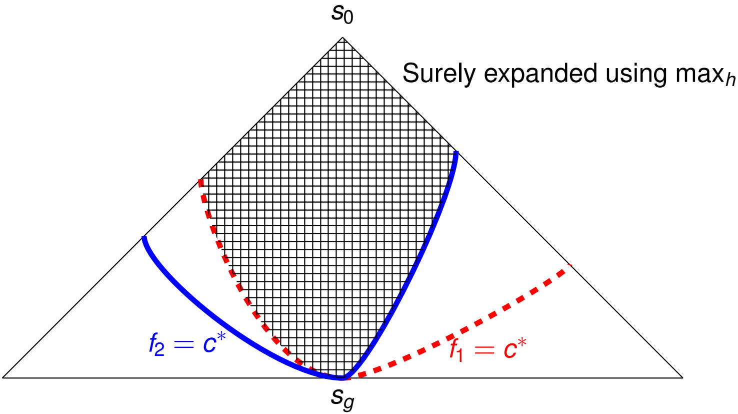

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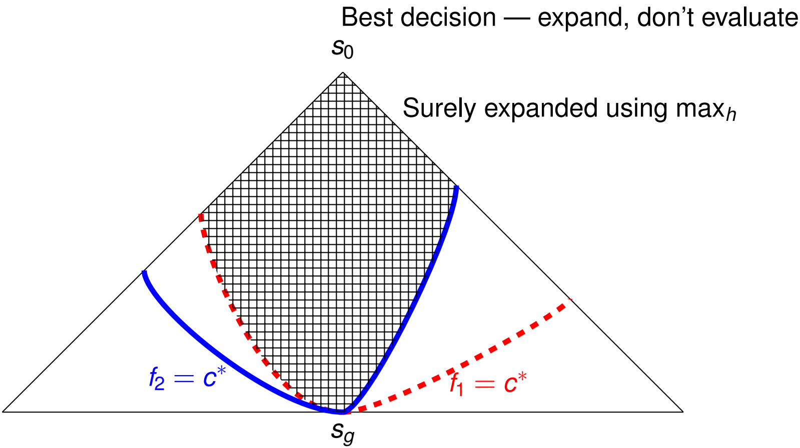

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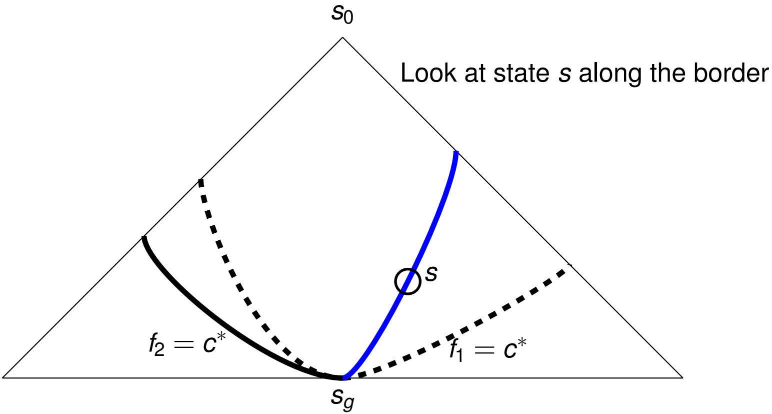

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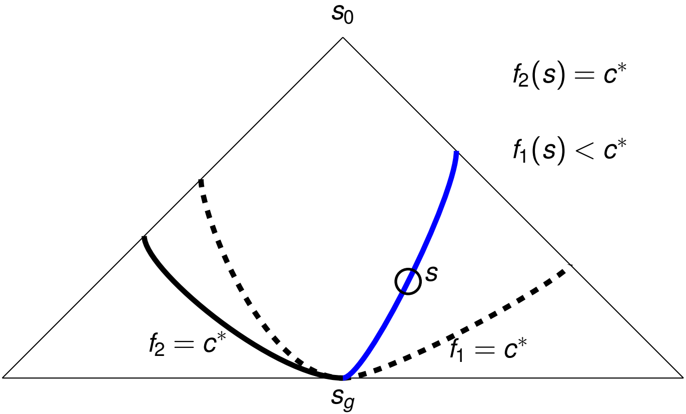

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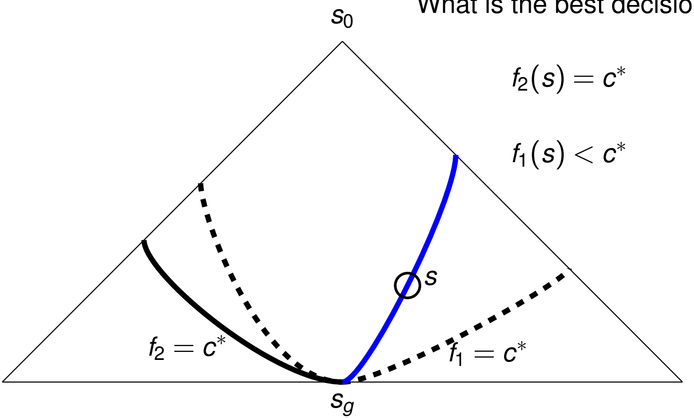

What is the best decision?

$f_2(s) = c^*$

$f_1(s) < c^*$

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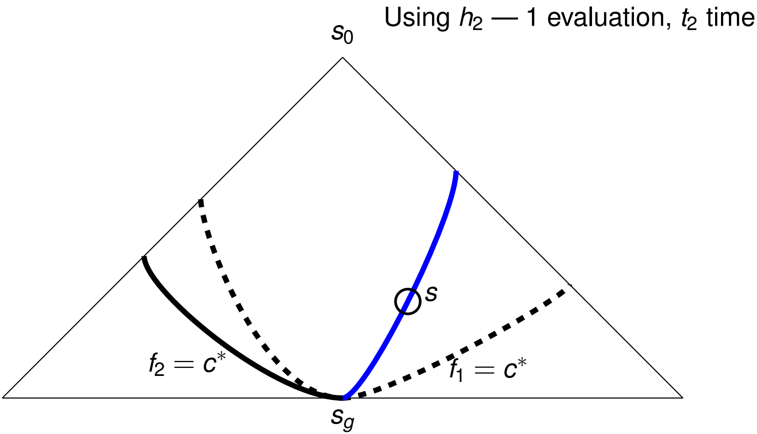
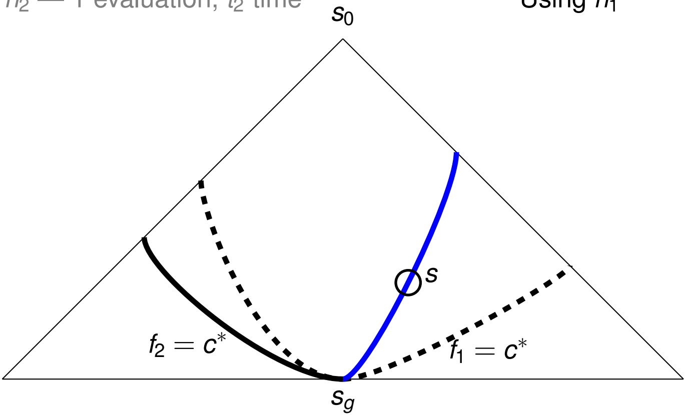


Theoretical Model - Which Heuristic to Compute When?

Using h_2 — 1 evaluation, t_2 time

Using h_1



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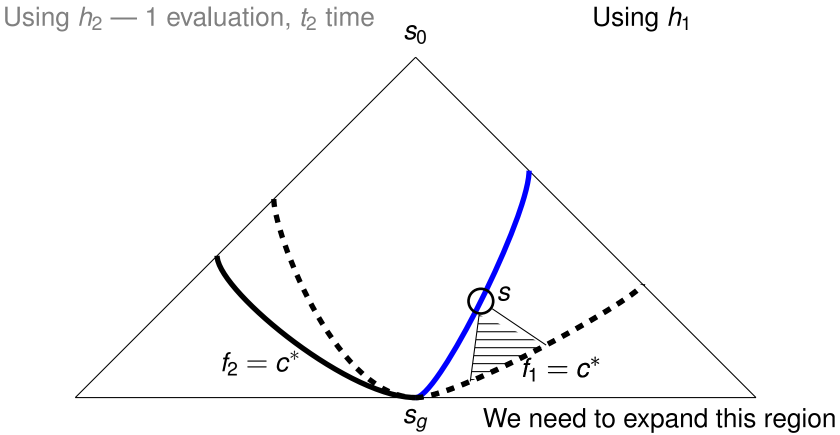

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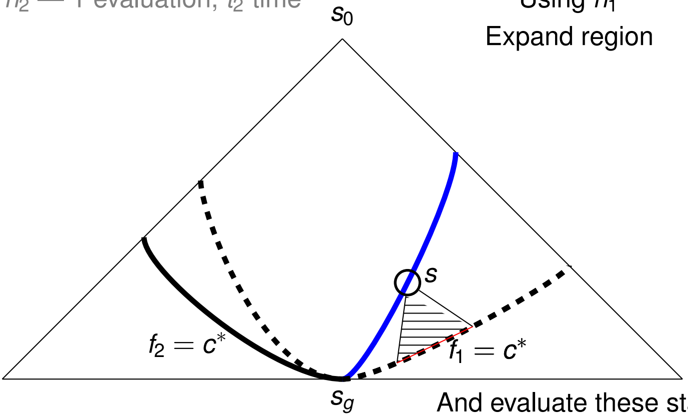
Expand region



And evaluate these states

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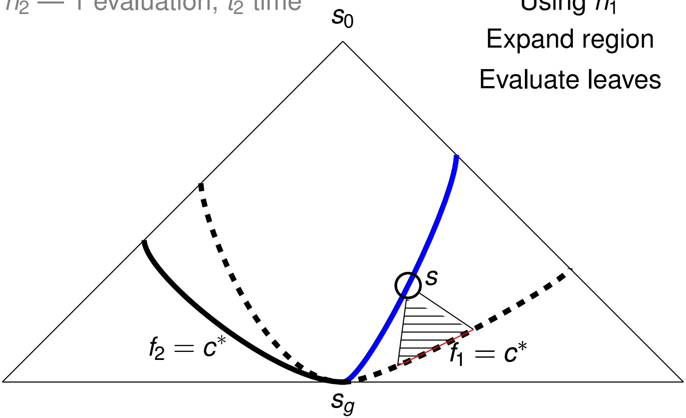
Expand region

Evaluate leaves



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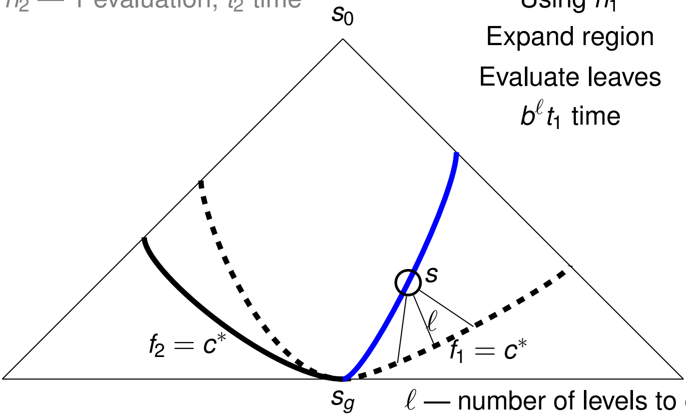

Using h_1

Expand region

Evaluate leaves

$b^\ell t_1$ time

ℓ — number of levels to expand

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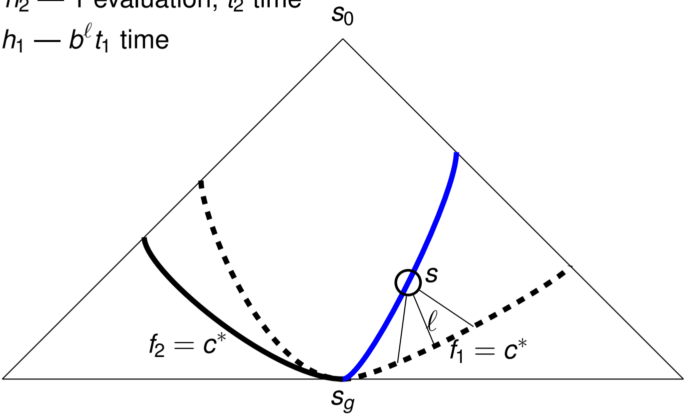

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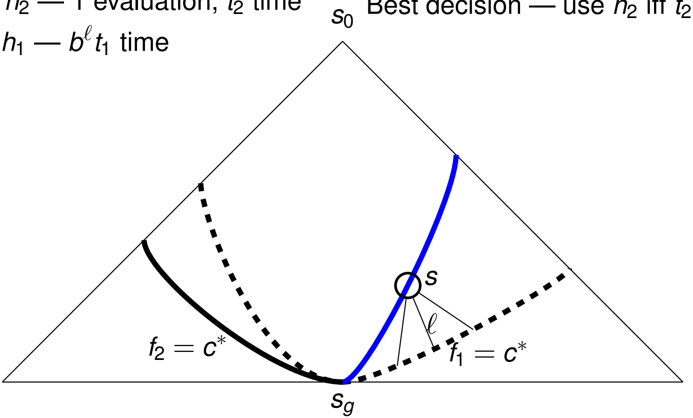
Best decision — use h_2 iff $t_2 < b^{\ell} t_1$



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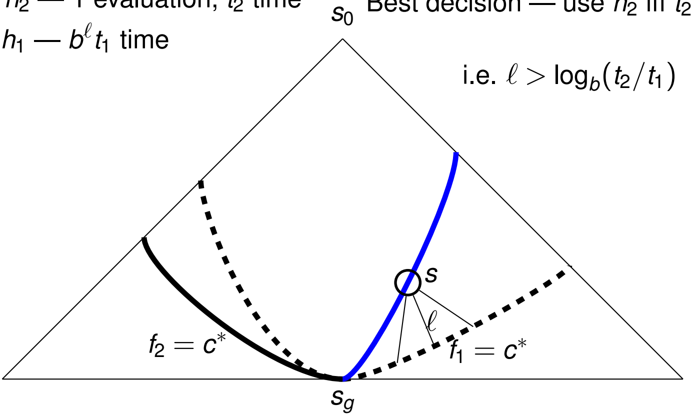
i.e. $\ell > \log_b(t_2/t_1)$



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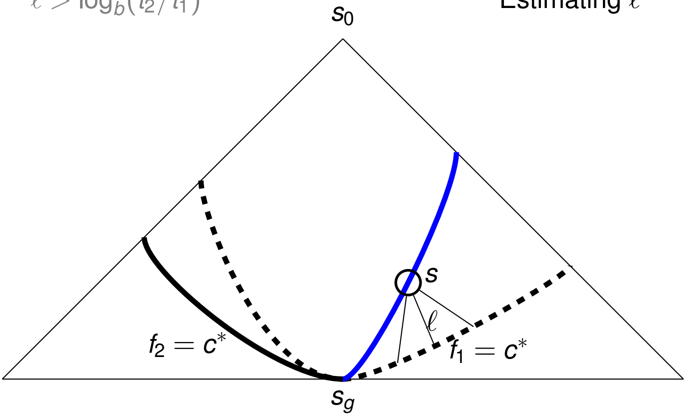

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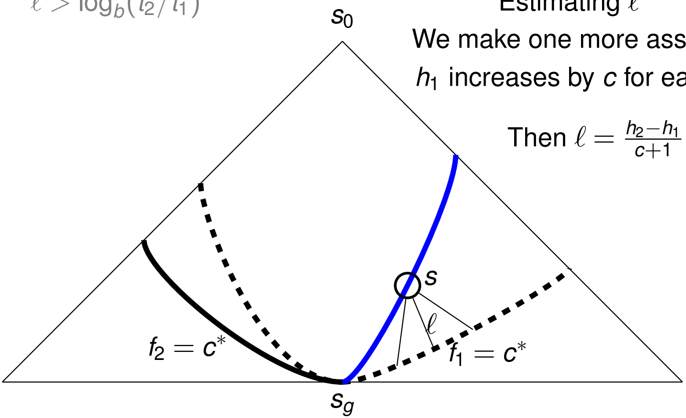


Estimating ℓ

We make one more assumption:

h_1 increases by c for each level.

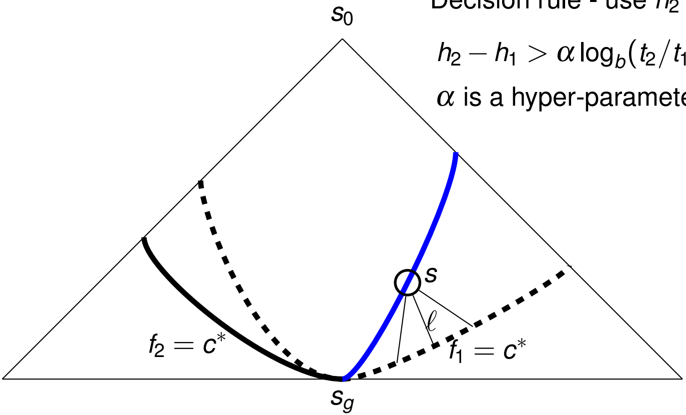
Then $\ell = \frac{h_2 - h_1}{c+1}$.

Theoretical Model - Which Heuristic to Compute When?

Decision rule - use h_2 iff

$$h_2 - h_1 > \alpha \log_b(t_2/t_1)$$

α is a hyper-parameter



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Two heuristics: h_1 and h_2

- Consistent
- Evaluating h_i takes time t_i

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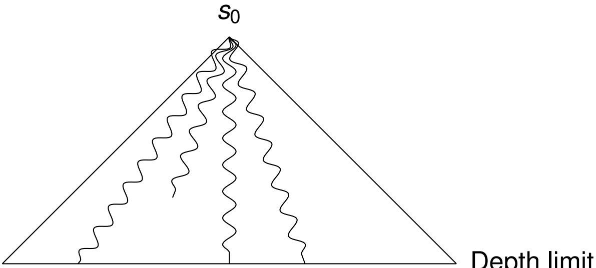
Learning

- Pre-search:
    - Collecting training examples
    - Labeling training examples
    - Generating features
    - Building a classifier
- During search:
    - Classification
    - Active learning

Collecting Training Examples

- State space is sampled using stochastic hill climbing “probes”
  - Depth limit = $2 * h(s_0)$
  - Probability of expanding successor $s \sim 1/h(s)$
- All *generated* states are added to the training set
- Probing stops when enough training examples are collected



Labeling Training Examples

- b, t_1, t_2 are estimated from the collected examples
- $h_2 - h_1$ is calculated for each state
- Each example is labeled by h_2 iff $h_2 - h_1 > \alpha \log_b(t_2/t_1)$

- WLOG $t_2 > t_1$ - the choice is always whether to evaluate the more expensive heuristic

Generating Features

- We perform online learning, for a specific problem, so we do not need to generalize across problems
- This allows us to use features which fully describe each state

- We use the simplest features - just values of state variables

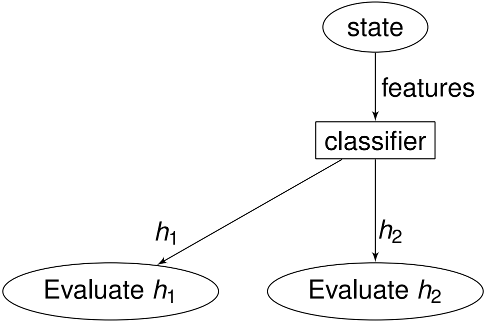
- Better features will probably lead to better results

Building a Classifier

- We use the Naive Bayes classifier
  - Very fast
  - Incremental — can be updated quickly on the fly
  - Provides probability distribution for classification

Using the classifier

State Evaluation



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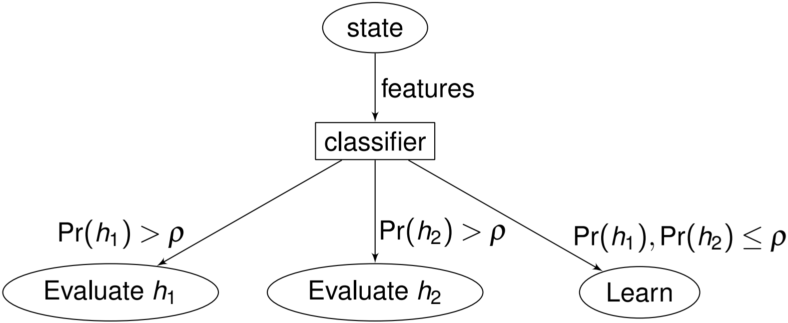


Final Remarks

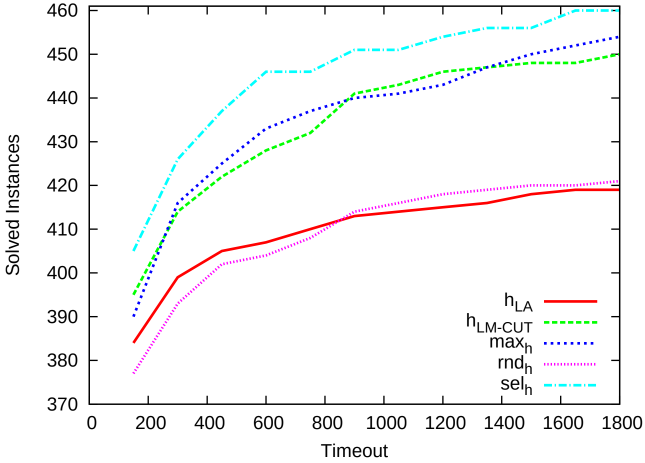
- This is an active online learning scheme
- Using multi-valued variable representation (and not STRIPS) helps, because it reduces dependence between state variables
- This approach can be easily extended to multiple heuristics
  - Learn a classifier for each pair
  - Decide which heuristic to use by voting

Outline

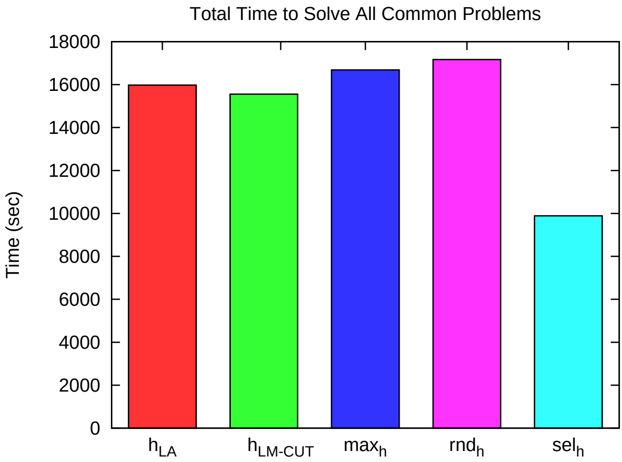
Evaluation

- We evaluted on problems from 22 domains from IPC 1 – 5
- We used two state of the art heuristics
  - $h_{\text{LM-CUT}}$ - Helmert and Domshlak 2009
  - h_{LA} - Karpas and Domshlak 2009
- Parameters
  - $\alpha = 1$ - decision rule bias
  - $\rho = 0.6$ - confidence threshold
  - Training set size = 100

Anytime Behavior



Results - Time



Conclusions

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- This leads to state-of-the-art performance

- Online learning can help in optimal planning

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