# Learning to Combine Admissible Heuristics Under Bounded Time

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## Motivation

- Optimal planning  $\equiv A^*$  + admissible heuristic (almost always)
- Which heuristic to use?
- Why use only one heuristic?
- Simplest combination method: max

Domain	$h_{LA}$	$h_{\text{LM-CUT}}$	max
airport	25	38	36
freecell	28	15	22

## The Problem with $\max$

- We need to compute many heuristic functions
  The heuristic value is the result of only one computation
- Some computation is wasted
- Possible solution: learn a classifier which predicts which heuristic will be the "winner".

Caveat: sometimes spending a lot of time to compute the most informed heuristic is not the best thing to do (see the results from the sequential optimal track in IPC-2008).

## **Theoretical Model**

#### Assumptions

- State space is a tree
- Single goal state
- Uniform cost actions
- Constant branching factor *b*
- Perfect knowledge
- Two heuristics:  $h_1$  and  $h_2$
- Consistent
- Evaluating  $h_i$  takes time  $t_i$

### **Decision Rule**

Above both lines is the surely expanded region. Best decision - just expand, don't evaluate.

For state *s* on the border, the options are either to use  $h_2$  which takes time  $t_2$ , or use  $h_1$ , in which case we need to expanded the high-lighted region, which take  $b^l t_1$  time.

Therefore the best decision for *s* is to use  $h_2$  iff  $t_2 < b^l t_1$ , or if  $l > \log_b(t_2/t_1)$ .

After some more assumptions about the rate of growth of heuristic error, we can write the rule as

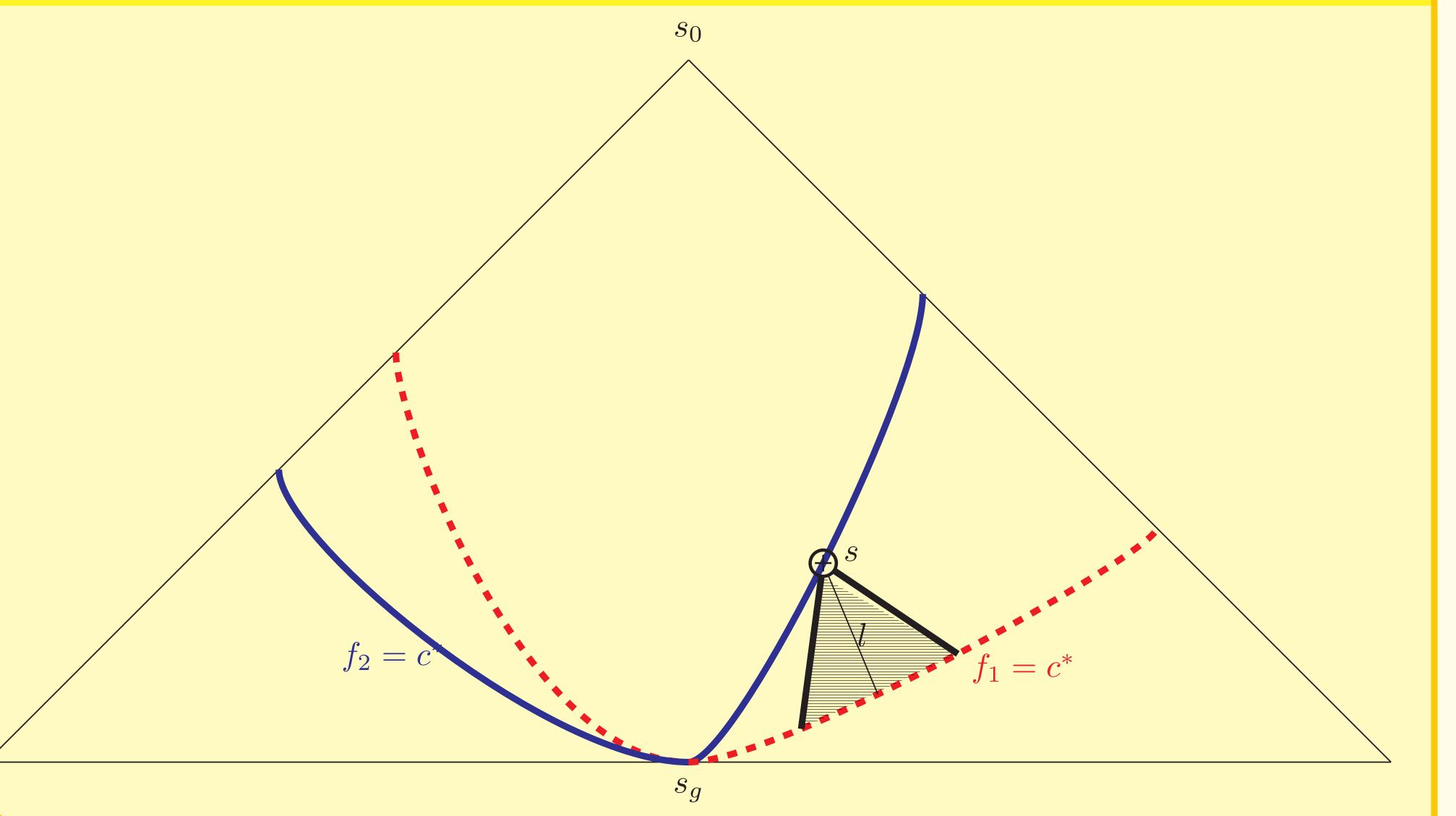
Use  $h_2$  iff  $h_2 - h_1 > \alpha \log_b(t_2/t_1)$ 

 $\alpha$  is a hyper-parameter

## **Dealing with Assumptions**

#### Assumptions

#### **Theoretical Model Illustrated**



- State space is a tree doesn't change the rule
- Single goal state doesn't change the rule
- Uniform cost actions doesn't change the rule
- Constant branching factor *b* estimate
- Perfect knowledge use decision rule at every state

Two heuristics:  $h_1$  and  $h_2$ 

- Consistent doesn't change the rule
- Evaluating  $h_i$  takes time  $t_i$  estimate

#### Learning

- Collecting training examples
- Labeling training examples
- Generating features
- Building a classifier

## **Collecting Examples**

State space is sampled using stochastic hill climbing "probes"

## Labelling Examples

First  $b, t_1, t_2$  are estimated from the collected examples. Then  $h_2-h_1$  is calculated for each state. Each example is labeled by  $h_2$  iff

 $h_2 - h_1 > \alpha \log_b(t_2/t_1).$ 

WLOG  $t_2 > t_1$  - the choice is always whether to evaluate the more expensive heuristic.

#### Features

We use the simplest features - values of state variables. Better features will probably lead to better results.

#### Classifier

We use the Naive Bayes classifier, because it is:Very fast

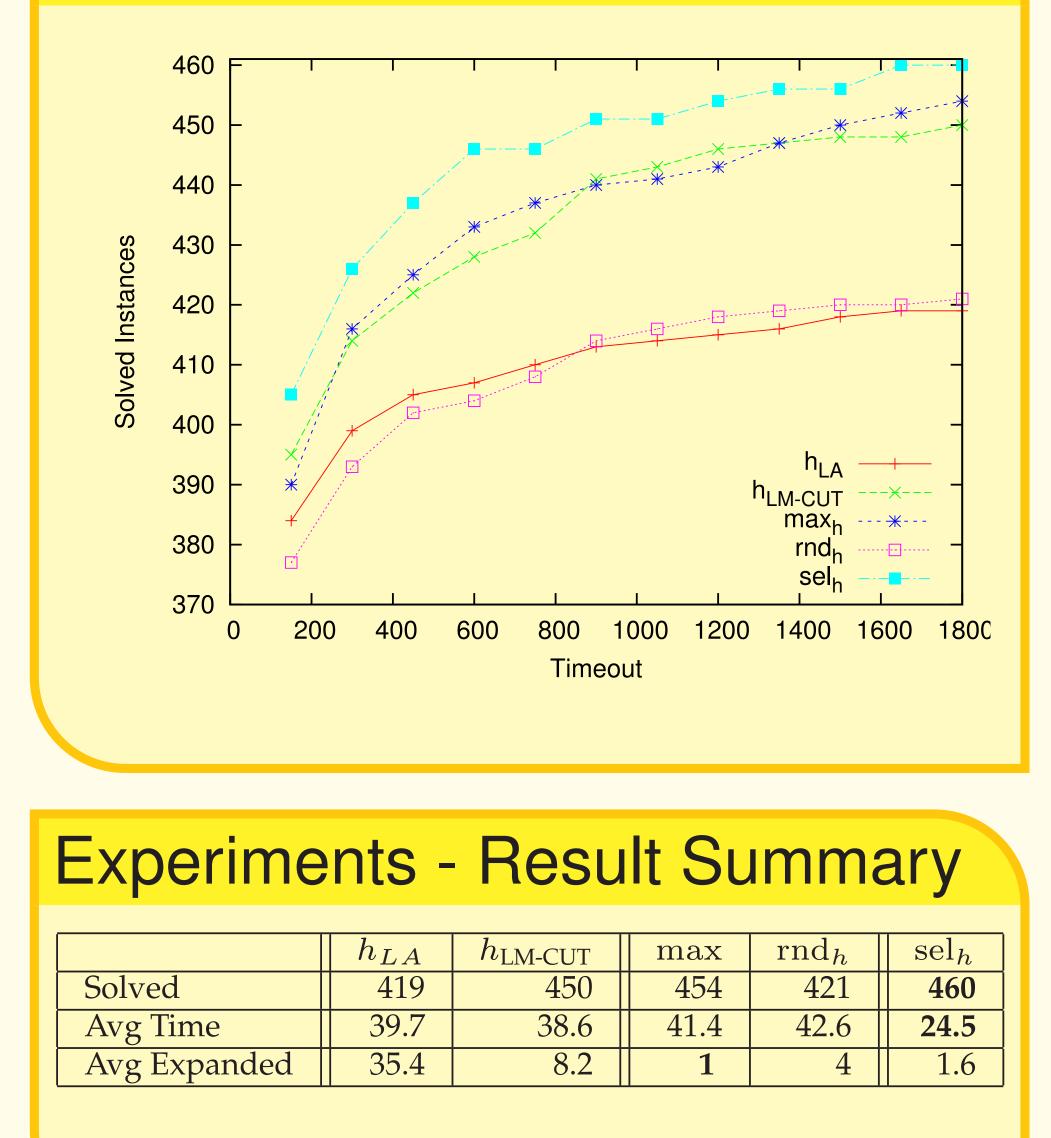
### Experiments

We used two state of the art heuristics:  $h_{\text{LM-CUT}}$  (Helmert and Domshlak 2009) and  $h_{LA}$  (Karpas and Domshlak 2009).

Parameters were set to  $\alpha = 1, \rho = 0.6$ , Training set size = 100.

We compare to each of the individual heuristics, regular max, and  $rnd_h$ , which selects one of the heuristics at random.

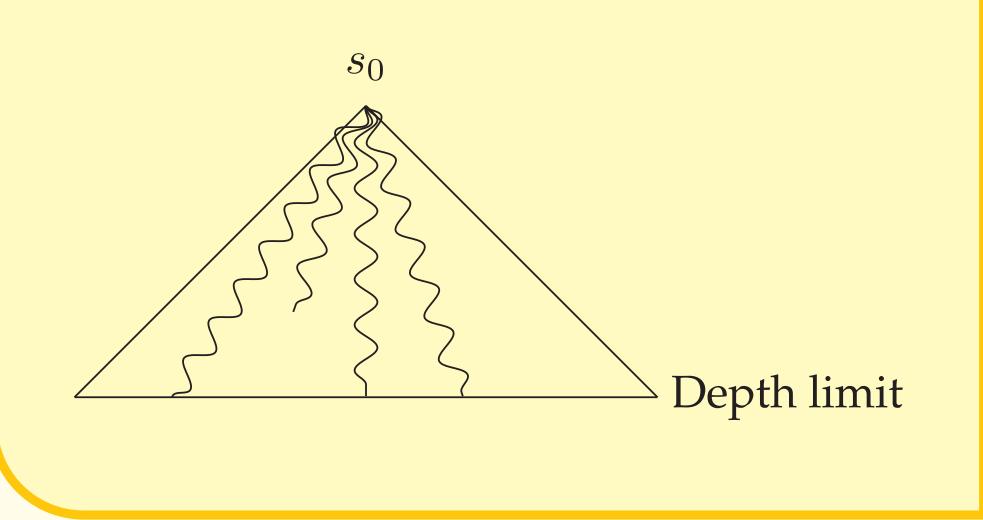
## **Experiments - Anytime Behavior**



• Depth limit =  $2 * h(s_0)$ 

• Probability of expanding successor s is 1/h(s)

All *generated* states are added to the training set. Probing stops when enough training examples are collected

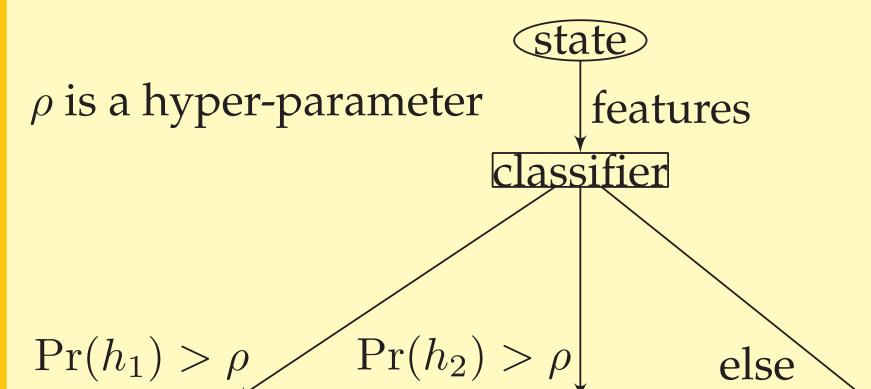


Incremental

Evaluate h

Provides probabilistic classification

Using the Learned Model



Evaluate  $h_2$ 

Learn